Radius of the Circumcircle of a Triangle

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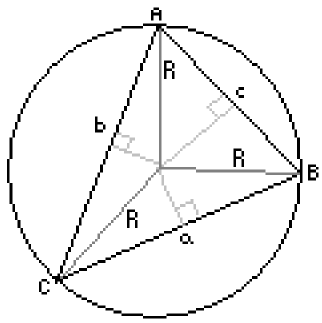

The center of the circumcircle of a triangle is located at the intersection of the perpendicular bisectors of the triangle. Given the side lengths of the triangle, it is possible to determine the radius of the circle.

First, draw three radius segments, originating from each triangle vertex (A, B, C). Then, draw the perpendicular bisectors, extending from the circumcenter to each side's midpoint (sides a, b, c). Three smaller isoceles triangles will be formed, with the altitude of each coinciding with the perpendicular bisector. Note that the vertex angles of the isoceles triangles subtend the same arc of the circumcircle as the angles of the larger triangle. Therefore, the measure of each vertex angle is twice that of its corresponding main angle. This means that the measures of the bisected vertex angles are exactly equal to the measures of the main angles. Start with the angle corresponding to angle A in one isoceles triangle:

$$\sin(A) = \frac{a/2}{R} \tag{1}$$

Using the fact that $1/2 \cdot bc \sin A = \Delta$, the triangle's area, we can rewrite the equation:

$$1/2 \cdot bc \sin(A) = \frac{abc}{4R} \tag{2}$$

$$\Delta = \frac{abc}{4R} \tag{3}$$

$$R = \frac{abc}{4\Delta} \tag{4}$$

The area of the triangle Δ can be determined by Heron's Area Formula, given the semiperimeter $s = \dfrac{a+b+c}{2}$:

$$\Delta \quad = \quad \sqrt{s\left(s-a\right)\left(s-b\right)\left(s-c\right)} \tag{5}$$