Roots of the Quadratic Equation

Brian Rogers

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A quadratic equation is a second-degree polynomial with two (not necessarily unique) solutions. To solve the equation in its general form, a technique known as **completing the square** is used. Completing the square involves converting an expression of the form $x^2 + kx$ to $(x + m)^2 + n$. This technique is employed in (4) below.

$$ax^2 + bx + c = 0 \tag{1}$$

$$a\left[x^2 + \frac{bx}{a} + \frac{c}{a}\right] = 0 \tag{2}$$

$$a\left[x^2 + \frac{bx}{a} + \frac{b^2}{4a^2} - \frac{b^2}{4a^2} + \frac{c}{a}\right] = 0 \tag{3}$$

$$a\left[\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + \frac{c}{a}\right] = 0 \tag{4}$$

$$a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c = 0 \tag{5}$$

$$a\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a} \tag{6}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2} \tag{7}$$

$$\left(x + \frac{b}{2a}\right) = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}} \tag{8}$$

$$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a} \tag{9}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{10}$$

There are two solutions, each corresponding to the positive or negative square root above. The term under the square root is known as the **discriminant**, $D = b^2 - 4ac$. The properties of the discriminant are summarized below.

If $D < 0$:	the quadratic has two non-real roots
If $D = 0$:	the quadratic has one real root
If $D > 0$:	the quadratic has two real roots